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Space-Time Structure Near Particles and Its Influence on Particle Behavior

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An interrelation between the properties of the space-time structure near moving particles and their dynamics is discussed. It is suggested that the space-time metric near particles becomes a curved one $\tilde{g}_{\mu\nu}(x, b_E)$ depending on a random vector $b_E = (b_4, \mathbf{b})$ with a distribution $w(b_E^2/l^2)$; the averaged space-time metric $\langle \tilde{g}_{\mu\nu}(x, b_E)\rangle$ over this distribution gives the general effect on particle behavior. As a result the particle motion in our scheme is described by a nonlinear equation. It turns out that the nonrelativistic limit of this equation gives a simple connection between the space-time structure at small distances and the dynamical behavior of particles. Different types of particle motion (nearly rectilinear, stochastic, and solitonlike) caused by some concrete forms of the averaged conformally flat space-time metric $\langle \tilde{g}_{\mu\nu}(x, b_E)\rangle$ are considered.

1. INTRODUCTION

In this paper we discuss the possibility of an interrelation between the properties of space-time structure and the dynamics of particles. This problem is not new to scientific literature. For example, Einstein (1924) thought to use the random fluctuations of the metric field $g_{\mu\nu}(x)$ as the origin of the real quantum forces which justify the stochastic interpretation of quantum mechanics. This idea was considered by Frederick (1976), Vigier (1982), and others (see the review due to Vigier, 1982). Moreover, an analogous idea on the question has been discussed by Efinger (1981), who wrote: "[If] freely moving particles are represented by solitary waves which, by definition, preserve their shape, then one could hypothesize that associated with these waves is a Riemannian metric $g_{\mu\nu}(x)$ which is nearly

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singular in a region corresponding to the width of the solitary wave-amplitude [an idea reminiscent of an old concept by Einstein, 1967]." According to this view, the particles in question are geometrical objects on a Riemannian space-time.

It is well known that in Einstein's theory of gravitation space-time structure near matter is changed and differs from flat space-time (for illustration see Figure 1). The validity of this assumption in a macroscale was demonstrated by the experiment measuring the deviation of light near the Sun. However, the question of what effect would be expected in the microworld is still open to discussion within this assumption, and in particular the influence of space-time structure near particles on their dynamic behavior.

We will base our study of this problem on the assumption that there exists a profound connection between the structure of space-time in the microworld and the propagation mechanism of particles. To realize this connection we assume that freely moving particles disturb space-time around themselves, and at the same time the space-time metric is generated and becomes a Riemannian one $\tilde{g}_{\mu\nu}(x, b_E)$ $(x = x_0, \mathbf{x})$ depending on a random four-vector $b_E = (b_4, \mathbf{b})$ with a distribution $w(b_E^2/l^2)$ obeying the conditions

$$w\left(b_E^2/l^2\right) \geqslant 0, \qquad \int d^4 b_E w\left(b_E^2/l^2\right) = 1,$$

$$\int d^4 b_E b_E^2 w\left(b_E^2/l^2\right) = l^2, \qquad \left(b_E^2 = b_4^2 + \mathbf{b}^2\right)$$

Parameter l characterizes the value (intensity) of fluctuations in metric and we call it the fundamental length.

Our next assumption is that a form of $\tilde{g}_{\mu\nu}(b_E, x)$ determines the space-time structure near particles and gives the general effect on the particle behavior. In other words, the character and type of the particle motion is essentially dependent on the space-time structure near moving particles. For example, we propose that the flat space-time structure gives rectilinear motion, and solitonlike, stochastic, and other types of particle motion may be caused by different forms of space-time structures around the particle. As a first step we choose a simple form

$$\tilde{g}_{\mu\nu} = \phi^2(x, b_E)\eta_{\mu\nu}$$

Then the averaged space-time metric acting on the behavior of particles is

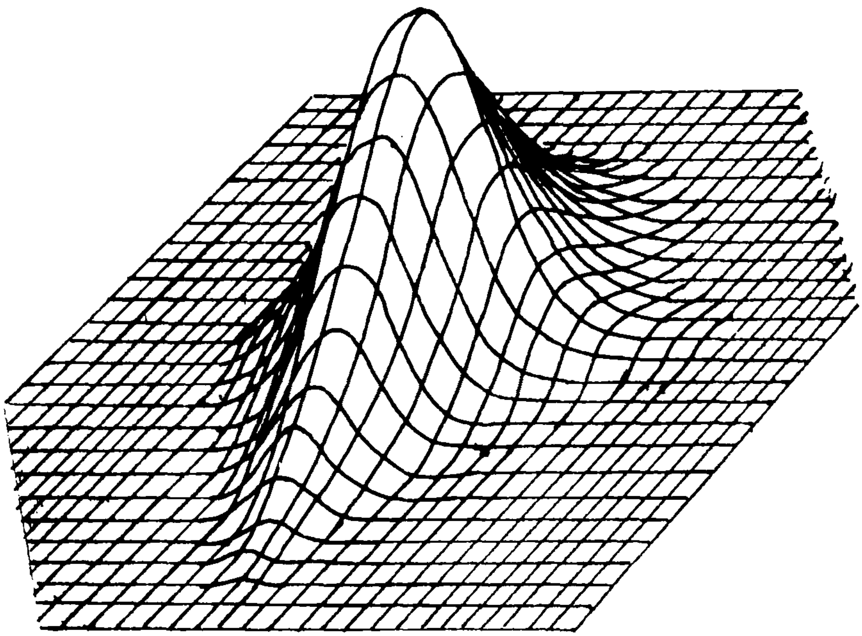

Fig. 1. The illustration of changing space-time structure near the particle.

obtained by the formula

$$g_{\mu\nu}(l, x) = \langle \tilde{g}_{\mu\nu}(b_E, x) \rangle$$
$$= \int d^4 b_E w(b_E^2/l^2)\phi^2(b_E, x)\eta_{\mu\nu} = \phi^2(l, x)\eta_{\mu\nu} \qquad (1)$$

where $\eta_{\mu\nu}$ is the Minkowski metric (for details, see Namsrai, 1984).

In this conformally flat space-time the action for a free particle acquires the form

$$S = -mc\int_a^b ds, \qquad ds = \left[g_{\mu\nu}(l, x)\, dx^\nu\, dx^\mu\right]^{1/2}$$
$$= \phi(x, l)\, ds_0, \qquad ds_0 = (dx^\nu\, dx_\nu)^{1/2} \qquad (2)$$

According to the action principle

$$dS = -mc\int_a^b \delta\, ds = -mc\int_a^b \delta\, ds^2/2\, ds = -mc\int_a^b \delta(g_{\mu\nu} dx^\nu dx^\mu)/2ds$$
$$= -mc\int_a^b \left\{ \frac{1}{2}\frac{dx^\nu}{ds}\frac{dx^\mu}{ds}\frac{\partial g_{\mu\nu}}{\partial x^\lambda}\delta x^\lambda + g_{\mu\nu}\frac{dx^\nu}{ds}\frac{d\delta x^\mu}{ds} \right\} ds = 0$$

we have the following nonlinear equation:

$$\frac{du_\nu}{ds_0} - \frac{\partial \ln \phi}{\partial x^\nu} + u_\nu \left(u^\lambda \frac{\partial \ln \phi}{\partial x^\lambda} \right) = 0 \tag{3}$$

Here $u^\lambda = dx^\lambda / ds_0$. Since $u^\nu u_\nu = 1$,

$$u^\nu \frac{du_\nu}{ds_0} = u^\nu \frac{\partial \ln \phi}{\partial x^\nu} - u^\lambda \frac{\partial \ln \phi}{\partial x^\lambda} \equiv 0$$

as is to be expected.

In a previous paper (Namsrai, 1984) we have attempted an explanation of the cosmic ray acceleration mechanism within this equation and have shown that the energy of a cosmic ray particle and its radius (the effective Schwarzschild), the age of Universe, and the value of the fundamental length l are connected with each other and are determined by a unified formula, Einstein's relation for the relativistic particle energy. From this we have obtained experimentally the value of the fundamental length $l = 1.56 \times 10^{-33}$ cm for the maximum proton energy observed in cosmic rays. The suggestion that the acceleration of cosmic rays is caused by the stochastic (or fluctuational) structure of space-time at small distances stimulated us to study the space-time structure near particles and its influence on particle behavior. Our aim is now to discuss equation (3) in the nonrelativistic limit by the appropriate choice of the form of the function $\phi(x, l)$. In conclusion of this section, we notice that the interrelation between space-time structure at small distances and the dynamic behavior of the particles appears at a deeper level and requires careful investigations. In this respect our approach is modest and belongs to the semi-empirical level.

2. STOCHASTIC BEHAVIOR OF PARTICLES AND ITS CONNECTION WITH STOCHASTIC MECHANICAL DYNAMICS

It is convenient to study the particle dynamic behavior arising from equation (3) and depending on the concrete form of the function $\phi(x, l)$ in the nonrelativistic limit. Before taking this limit for equation (3) we make some remarks. Generally speaking, according to the hypothesis of curved space-time structure around particles, the concept of their trajectory and velocity should be changed and generalized appropriately. In other words, it is quite possible that because of the metric fluctuations in the presence of particles, the description of the behavior of particles has a universal character and requires probabilistic methods. In particular, the particle velocity depends not only on the time variable t but also on the spatial variables $\mathbf{x}$: $\mathbf{v}(t) \to \mathbf{v}(\mathbf{x}, t)$. We suggest that for a complete description of the particle motion within our approach, together with the particle velocity $\mathbf{v}(\mathbf{x}, t)$ one should introduce two more quantities: $\rho(\mathbf{x}, t)$, the probability density of finding the particle at point $\mathbf{x}$ and at time t; and $\mathbf{u}(\mathbf{x}, t) = D\nabla \ln \rho(\mathbf{x}, t)$, the stochastic velocity, where D is some constant physical meaning, which will be discussed below. Thus, our basic idea is the following: fluctuations in metric take place everywhere (increasing in the presence of the particles), and may play the role of origin of the real "quantum" forces and lead to the random behavior of the particles; their dynamics are described by nonlinear partial differential equation of the type (3) admitting random solutions, and by the equation of continuity for $\rho(\mathbf{x}, t)$:

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) = 0 \tag{4}$$

In this case the particle velocity is given by the formula

$$\mathbf{v}(t) = \int \rho(\mathbf{x}, t)\mathbf{v}(\mathbf{x}, t)\, d^3x$$

In order to obtain the nonrelativistic equation of motion in our approach we go over to the formal limit $c \to \infty$ in equation (3). The time component of this equation in the case of $c \to \infty$ acquires the form

$$\mathbf{v} \cdot \nabla \ln \phi = 0 \tag{5}$$

The passage to the limit $c \to \infty$ in the spatial components of (3) gives the following relation:

$$\frac{1}{2}\frac{\partial}{\partial t}\mathbf{v}^2 + \mathbf{v}^2 \frac{\partial \ln \phi}{\partial t} = 0 \tag{6}$$

taking into account the relation (5). From this equation we see that if the function ϕ does not depend on the time variable then the particle velocity is constant and the particle moves along a rectilinear trajectory. This situation also takes place in the limit, when the value of the fundamental length l is neglected, and thus, by our assumption, $\phi(x, l) = 1$ at $l \to 0$.

After simple integration of (6) we have

$$v^2\phi^2 = \text{const} \tag{7a}$$

Here the constant may be obtained from an initial condition, say, $v^2\phi^2|_{t=0}$

$= v_0^2$. Thus, the particle velocity $\mathbf{v}(\mathbf{x}, t)$ and the form of the function $\phi(x, l)$ are related by the formula

$$\mathbf{v}^2 = \mathbf{v}_0^2/\phi^2 \tag{7b}$$

Physically this relationship means that knowing the space-time structure near the particle we can calculate its velocity (generalized) and conversely, knowing the value of the particle velocity we can build the space-time structure near the moving particle. Thus, it seems, there exists a profound connection between these two concepts and they enter as a single entity inseparably into our scheme. Therefore, generally speaking, the function $\phi(x, l)$ should be dependent on the properties of the particle: on its mass, an effective size λ (in the quantum mechanical case $\lambda = \hbar/mc$) and the initial velocity v_0, and so on. In this paper, we consider three cases for the function $\phi(x, l) \Rightarrow \phi(x, t; l, \lambda, v_0)$ (here the space-time dimension is two: x, t):

I. $\phi_1(x, t; l, \lambda, v_0) \to \infty$ for $t \to \infty$

$$\phi_1(x, t; l, \lambda, v_0) \to 0 \quad \text{at } |x| \to \infty \tag{8}$$

II. $$\phi_2(x, t; l, \lambda, v_0) \to 1, \quad \text{for } t, |x| \to \infty \tag{9}$$

III. $$\phi_3(x, t; l, \lambda, v_0) \to \infty \quad \text{for } t, |x| \to \infty \tag{10}$$

We assume here the equality $\phi_i(x, t; l, \lambda, v_0) = 1$ at $l \to 0$, is fulfilled for all three cases. Now we consider the first case and the simple form

$$\phi_1(x, t; l, \lambda, v_0) = (1 + bt^2)/[1 + (b/v_0)tx] \tag{11}$$

This gives

$$v = (v_0 + btx)/(1 + bt^2) \tag{12}$$

where $b = 4D^2(l/\lambda^3)$, and D is a universal constant dimension of [cm^2/sec], $D \sim \hbar/m$. The solution of the equation of continuity (4) with the velocity (12) and the initial condition

$$\rho(x, t = 0) = a^{-1}\pi^{-1/2}\exp(-x^2/a^2)$$

has the form

$$\rho(x, t) = a^{-1}\pi^{-1/2}(1 + bt^2)^{-1/2}\exp\left[-(x - v_0 t)^2/a^2(1 + bt^2)\right] \tag{13}$$

where $a=(l/\lambda^3)^{-1/2}$. According to the above deduction in our case the stochastic velocity of the particle is given by

$$u(x,t)=D\nabla \ln\rho(x,t)=-b^{1/2}(x-v_0t)/(1+bt^2) \tag{14}$$

The mean value and dispersion of the quantities $v(x,t)$ and $u(x,t)$ are determined by the formulas

$$\langle v(x,t)\rangle=\int dx\, v(x,t)\rho(x,t)=v_0$$

$$\langle u(x,t)\rangle=\int dx\, u(x,t)\rho(x,t)=0$$

$$\text{Dis.}v^2(x,t)=\tfrac{1}{2}a^2b^2t^2/(1+bt^2) \tag{15}$$

Moreover, by definition

$$\langle X(t)\rangle=\int dx\, x\rho(x,t)=v_0t$$

$$\text{Dis.}X^2(t)=\tfrac{1}{2}a^2(1+bt^2)$$

$$\langle v^2(x,t)\rangle=v_0^2+\tfrac{1}{2}a^2b^2t^2/(1+bt^2) \tag{16}$$

From the formulas (12)–(16) we conclude immediately that the simple space-time structure near moving particles determined by the expression (11) gives completely the stochastic mechanical results obtained by Namsrai (1981) if the constant D is taken equal to the diffusion coefficient $\nu=\hbar/2m$. In this case the space-time metric near stochastic particles takes the form

$$ds^2=\phi_1^2(x,t;l,\lambda,v_0)(c^2dt^2-dx^2) \tag{17}$$

where $\phi_1(x,t;l,\lambda,v_0)$ is given by the formula (11).

It should be noted that this metrical form gives rise to a random behavior of particles (like Brownian motion), i.e., the particle is forced to move stochastically during its propagation in space-time. But, on the other hand, particles always take care of the space-time structure around themselves in order to move further. This is essentially a dialectical unity of motion and space-time.

3. SOLITONLIKE BEHAVIOR OF PARTICLES

Now we consider the cases (9) and (10), and choose the following simple forms:

$$\phi_2(x, t; l, \lambda, v_0) = 1 + l^2/\left[\lambda^2 + (x - v_0 t)^2\right] \tag{18}$$

$$\phi_3(x, t; l, \lambda, v_0) = \left(1 + \frac{l}{\lambda}\right)\frac{\sinh b}{\cosh^3 b}\cosh^3\left(l\frac{x - v_0 t}{\lambda^2} + b\right)\sinh^{-1}\left(l\frac{x - v_0 t}{\lambda^2} + b\right) \tag{19}$$

where b is some positive constant. These expressions satisfy the conditions (9), (10), and $\phi_i = 1$ $(i = 2, 3)$ at $l = 0$, for any x and t. The following generalized velocities of the particle:

$$v_2 = v_0/\phi_2 = v_0\frac{\lambda^2 + (x - v_0 t)^2}{\lambda^2 + l^2 + (x - v_0 t)^2}$$

and

$$v_3 = v_0/\phi_3$$

correspond to the formulas (18) and (19), respectively. From these velocities v_2 and v_3 we have generalized "trajectories":

$$\left[v_0 t - X_2(x, t)\right] = \frac{l^2}{(\lambda^2 + l^2)^{1/2}} \times \arctan\left\{\frac{v_0 t}{(\lambda^2 + l^2)^{1/2}}\frac{1}{1 + \left[x(x - v_0 t)/(\lambda^2 + l^2)\right]}\right\} \tag{20}$$

$$X_3(x, t) = \frac{\lambda^2}{2l}\left(1 + \frac{l}{\lambda}\right)^{-1}\cosh^3 b \sinh^{-1} b \times\left\{\operatorname{sech}^2\left[\frac{l}{\lambda^2}(x - v_0 t) + b\right] - \operatorname{sech}^2\left(\frac{lx}{\lambda^2} + b\right)\right\} \tag{21}$$

It is easy to verify that

$$X_2(x, t) \to x_2(t) = v_0 t$$

$$X_3(x, t) \to x_3(t) = v_0 t$$

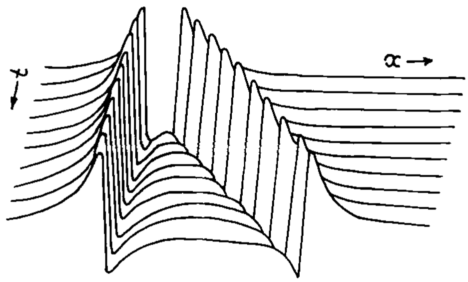

Fig. 2. Solitary-type wave corresponding to the formula (20). We see here two precise maximums-wave, the right part of which moves at a constant velocity without change of shape, while the left part remains near the point $x = 0$. From this plot and the formula (20) one can suggest that in nature there may exist some type of particle motion which consists of two parts: rectilinear and solitonlike; however, because of the small value of the fundamental length l the latter type motion is not observed in practice.

in the limit $l \to 0$. Behaviors of the functions $[v_0 t - X_2(x, t)]/l$ and $X_3(x, t)/l$ are shown in Figures 2 and 3, respectively. From these figures we observe that the "trajectories" obtained by the functions ϕ_2 and ϕ_3 correspond to a solitonlike motion of the particle. However, in the case of (20) because of the small value of the fundamental length l we do not, in fact, observe a deviation from the rectilinear particle trajectory for any value of λ, even up to $\lambda \sim l$. But the situation is different in the case of (21). When $\lambda \lesssim 10^{-15} \div 10^{-16}$ cm, the amplitude of the process (21) is of order unity, and therefore we can, in principle, experimentally observe solitonlike particle behavior caused by the space-time structure near the moving particle.

It should be noted that case (8) differs essentially from the other two cases (9) and (10). In the first case the generalized trajectory $X_1(x, t)[v_1(x, t) = \dot{X}_1(x, t)]$ corresponding to the velocity $v_1(x, t)$ becomes infinite at $x \Rightarrow \pm\infty$, i.e., the concept of trajectory in this case loses its significance and has no physical meaning, while in the other cases the functions $[v_0 t - X_2(x, t)]$ and $X_3(x, t)$ are located along the rectilinear classical trajectory $x(t) = v_0 t$.

Finally, for illustration, the space-time structure $ds^2 = \phi^2(x, t) \times \eta_{\mu\nu}\, dx^\nu\, dx^\mu$, i.e., the function $\phi^2(x, t)$ corresponding to the particle behavior determined by the formula (21), is shown in Figure 4. In our semiempirical approach the question of a possible unique choice of the functions $\phi(x, t; l, \lambda, v_0)$ is not answered and seems to require another fundamental physical principle. At present, the formulation of this principle is not known and needs deeper study. However, we assume that this important problem may be solved alternatively, namely, by the character of the particle motion.

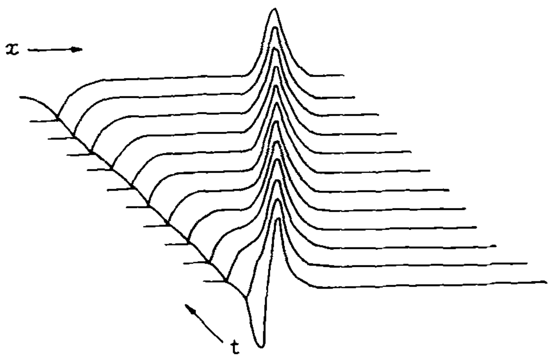

Fig. 3. Behavior of the "trajectory" (21) at the value of $\lambda = 1$. This plot is a hump-shaped wave exactly as Scott Russell observed (see Dodd et al., 1983), which moves along the classical trajectory $x = v_0 t$ at a constant velocity without change of shape. For larger values of λ the wave is broader and higher, but becomes thinner and short the smaller λ becomes. The left part of this figure, it appears, has no physical meaning. It is caused by the initial condition of the problem and corresponds to some minimum located near the point $x = 0$. This motionless minimum gives a "canal" over the time variable, a plot of which is not seen in the figure.

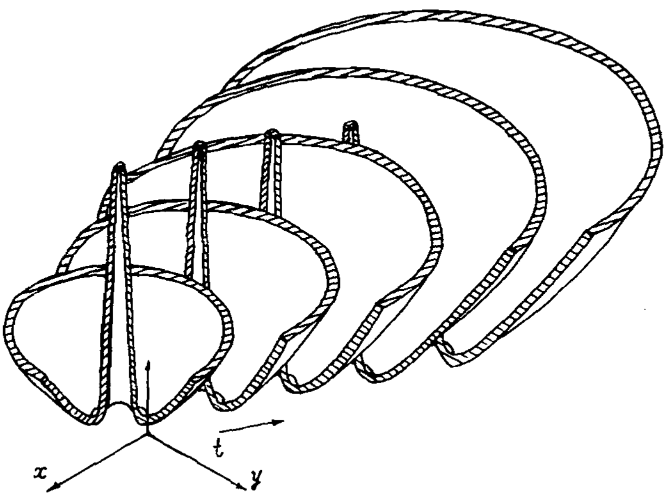

Fig. 4. A plot of the space-time structure, i.e., $\phi^2(x, t)$ corresponding to the solitonlike motion sketched in Figure 3. From this diagram we see that the metric $ds^2 = \phi^2(x, t) ds_0^2$ in the conformally flat space-time is singular in the region corresponding to the classical trajectory $x = v_0 t$. According to the diagram one can conclude that in order to hold a particle in a localized region during its propagation in space-time, or, for energy to be propagated in localized stable "packets" without being dispersed, it needs enormous efforts for space-time, i.e., space-time sets up an infinitely higher barrier wall around a particle.

This means that one can try to construct the space-time structure near the moving particle by means of the value of its velocity. For example, if we observe a rectilinear trajectory for the particle, we can conclude that the space-time structure near such a particle is flat. The role of that particle may be played by a photon, since, according to the relation (7^a), if the constant in the right-hand side of (7^a) is equal to the velocity of light c then the current velocity $v(x, t)$ should be equal to c, and therefore $\phi = 1$ everywhere for all t. We propose here that the light velocity in space-time is truly constant. If this assumption is valid at small distances (or at very-high energies) then light does not disturb space-time around itself, which always remains flat. In contradistinction to rectilinear motion, other types of particle motion correspond to a curved space-time structure.

In conclusion, we notice that metric fluctuations which give rise to stochastic and other types of particle motion, if they exist, would be detected by ultrahigh energy particles if their wavelength λ were comparable to or shorter than the value of the fundamental length $l \sim 10^{-33}$ cm. In other words, the properties of a particle with mass $m \ll M = \hbar/lc \sim 10^{-5}$ g ($\lambda = \hbar/mc \to \infty$) would be essentially insensitive to the fluctuational structure of space-time.

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